Planar three-body scattering of near-gravitational potentials

Morgan Long and Brian Stewart, Department of Physics

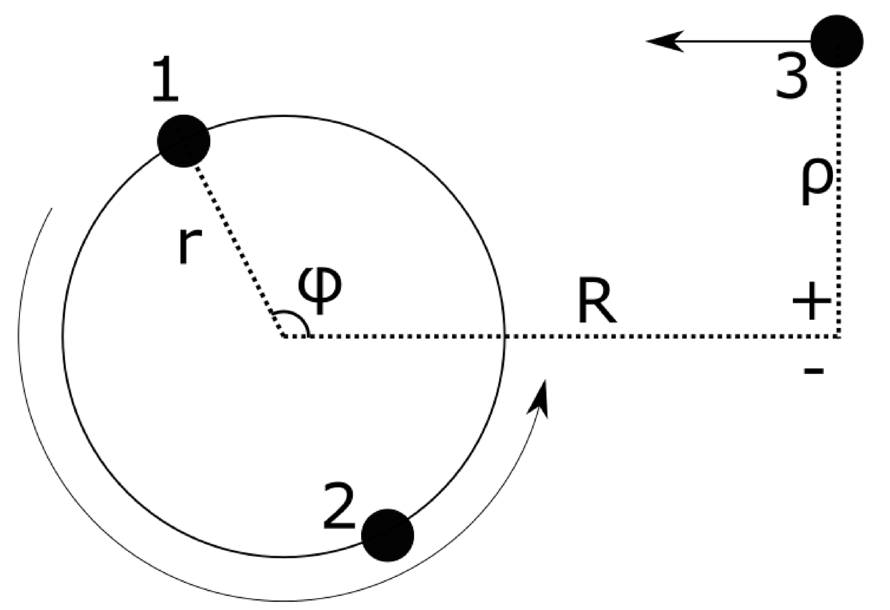
Background

The N-body problem is to predict the trajectories of N particles interacting under a specific potential, traditionally the gravitational potential, V_G. This problem is notoriously complicated for $N > 2$. We are specifically interested in the three-body planar scattering problem (3-BPSP). Initially, a pair of bodies orbit each other in a circle. A third approaches and interacts with the pair. All three bodies are restricted to a single plane, making the motions two-dimensional.

Much work has been done investigating the gravitational 3-BPSP. V_G is very similar to the attractive term used in many molecular potentials V_n. This work aims to connect gravitational studies to those of molecules.

$$V_G = -\sum_{j \neq i} \frac{G m_i m_j}{r_{ij}} \qquad V_n = -\sum_{j \neq i} \frac{G m_i m_j}{r_{ij}^n}$$

Methods

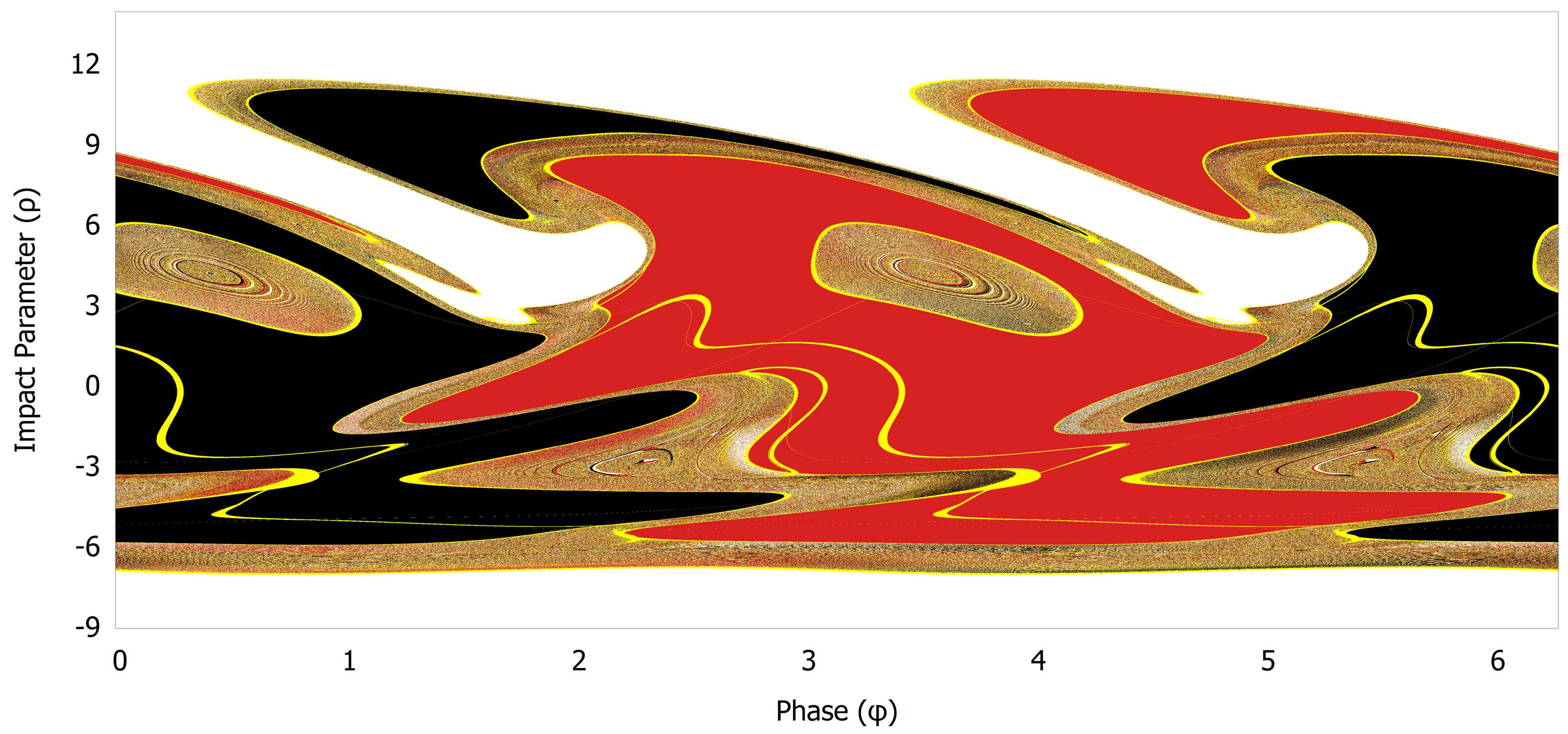


For our simulations, $G = 1$, $m_1 = m_2 = m_3 = 1$, $r = 1$, and $R = 160$. The velocity of the impacting body is written in terms of the critical velocity v_c. This is the unique velocity for the impacting body at which the whole system has zero energy. Our figures are at $\frac{1}{2}v_c$.

In our numerical simulations, we have two initial conditions, ϕ, ρ. ϕ is the initial phase of the rotating pair, and ρ is the impact parameter. In our plots, each pixel corresponds to a different set of initial conditions. The color codes the final outcome. White is a flyby, meaning the initially orbiting pair remain orbiting at the end. Black and red are different types of exchanges, meaning one of pair and the impacting body remain orbiting at the end. Finally, yellow means that either the simulation took too long to finish, or has lost too much precision.

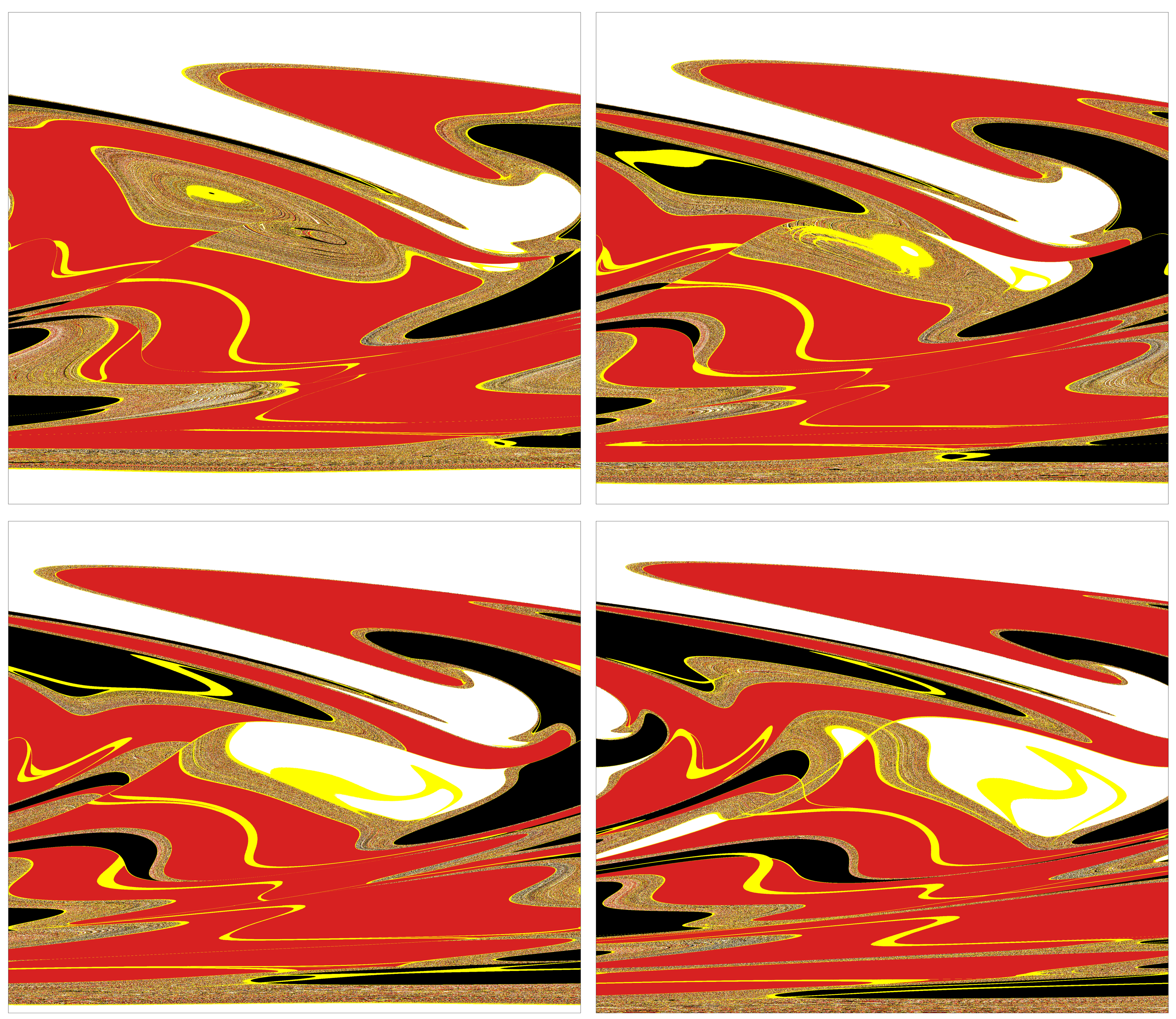
Near Gravitational Potentials

Yellow ribbons run across the gravitational plot, some of which are quite thin. These ribbons correspond to head-on impacts, which cause loss of precision. For gravity, a regularization allows head-on collisions to occur without loss.



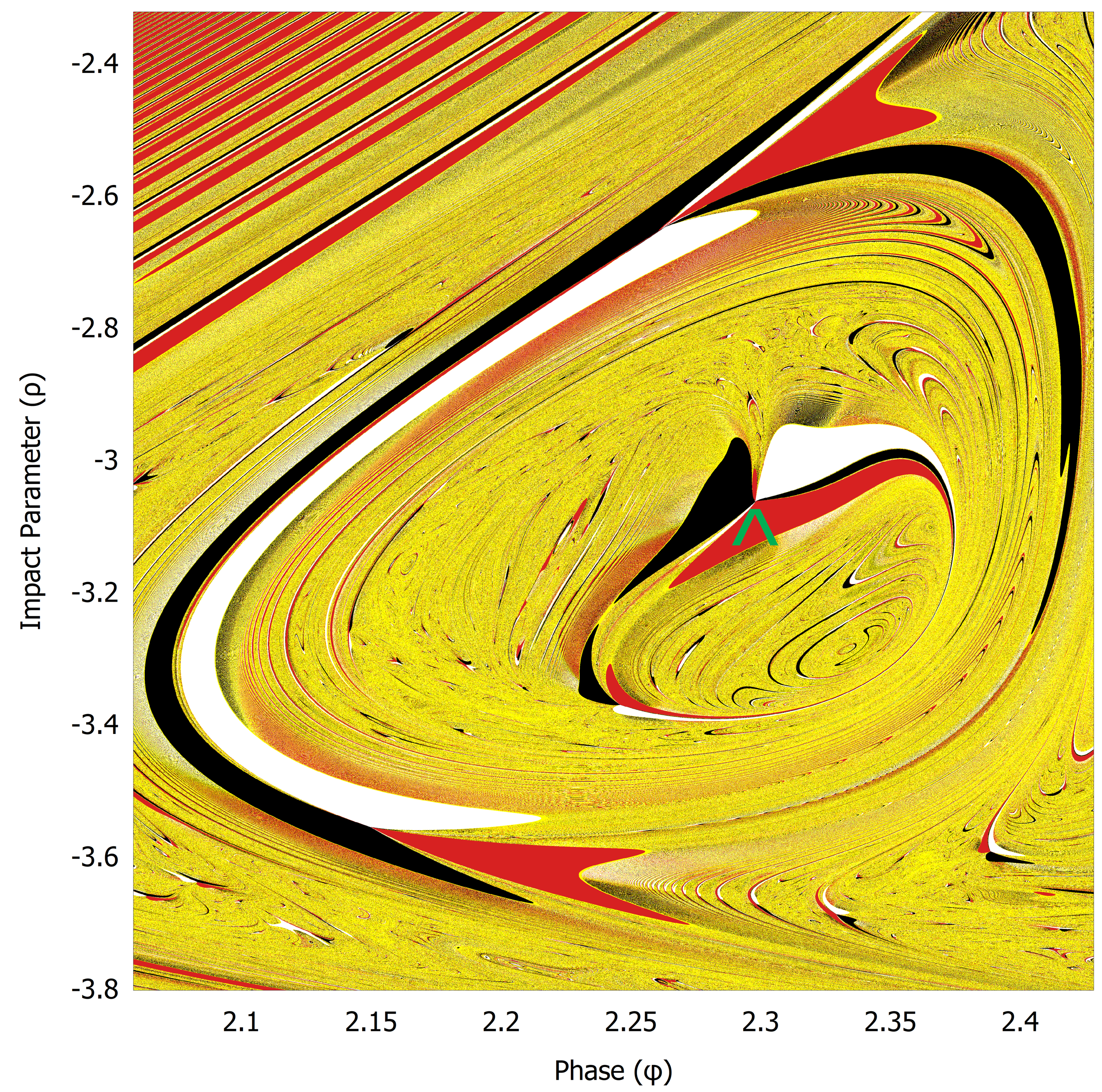
Final outcomes for gravity, n=1.

For $n \approx 1$, the $\phi - \rho$ plots shear along the the ribbons, causing discontinuities in the regions. The further from $n = 1$, the greater the shearing.



The final outcomes for a few potentials of the form V_n. Axis are as above, with only π phase width. Left to right, top to bottom: n = 1.05, 1.1, 1.15, 1.2

Triple Impacts



Head-on impacts between all three bodies, and collisions which last an infinite times are the only non-integrable points. We suspect that at the points where white, black, and red all touch are initial conditions where triple impacts occurs at some time during the collision. Based on other data, the three bodies get very close at the center most tri-color region, in particular.

Future Work

1. We plan to improve our simulations by coding for GPU's, and using quadruple precision. are working to achieve a deeper theoretical understanding of these images
2. More work needs to be done to precisely locate triple impacts, and infinite duration collisions.

References

[1] Joerg Waldvogel,
"A New Regularization of the Planar Problem of Three Bodies",
Celestial Mechanics, **006** (1972).

[2] Piet Hut,
"The Topology of Three-Body Scattering",
The Astronomical Journal **88** (1983).

[3] Aaron Larner,
Comparison of Molecular and Gravitational Three-Body Collisions,
Wesleyan Honors Thesis (2008).