Large Solid-Body Rotation Rates in Turbulent Flows

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Introduction

The orientation of large anisotropic particles advected in a fluid flow is a critical component of the study of fluid dynamics. The orientation has many applications in a variety of areas, such as plastic transport in the ocean. Particles smaller than the Kolmogorov length scale η act as tracers; however, tracer particles can reveal only so much about the flow structure. Particle alignment also poses a fascinating problem in physics: how do particles align themselves in turbulent flows and how is that alignment affected by their shape and orientation?

Point-like particle transport and mixing in turbulence has been meticulously studied and used to research vorticity dynamics and its correlation with other turbulent flow qualities. For example, research with tracer particles has revealed that fluid elements decelerate faster than they accelerate. This appears as flight crash events, which reveal the irreversible nature of chaotic flows. A natural extension of this work is to investigate the motions of larger, non-point-like particles and their alignment with the flow.

Triads and Tetrads

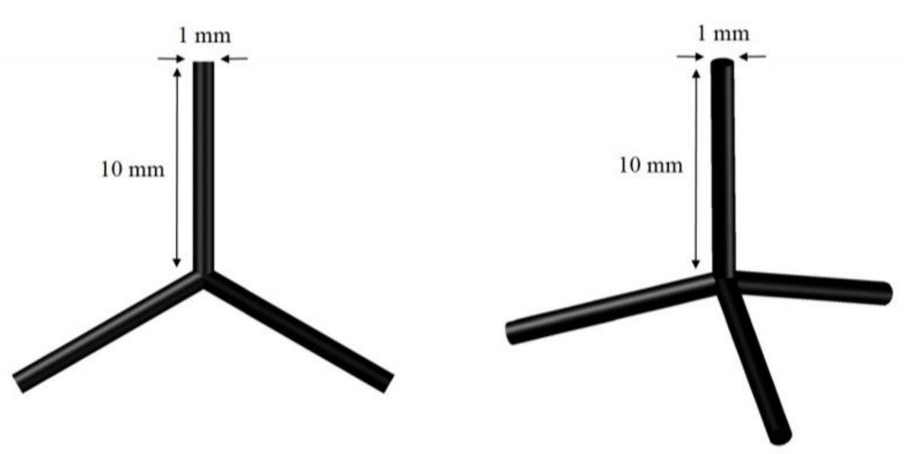


Figure 1: Particles above the Kolmogorov scale behave identically to a corresponding ellipsoid with identical mass and volume. (a) 3-armed triad, corresponding to a disc shape. (b) 4-armed tetrad corresponding to a spherical shape.

Pertinent Directions of Particles

All spheroidal particles have an orientation p, about which the particle is rotationally symmetric. They move with a translational acceleration a, and rotate with a rotation rate Ω and angular acceleration α. Translational velocity is not a relevant value in these dynamics. These components are vital to our understanding of particle dynamics in strain flow structures in turbulence. Since spherical particles are isotropic, their p is degenerate and thus it cannot align with any of the other pertinent vectors. By contrast, disc-shaped particles tend to align their p parallel to their a and perpendicular to their Ω and α.

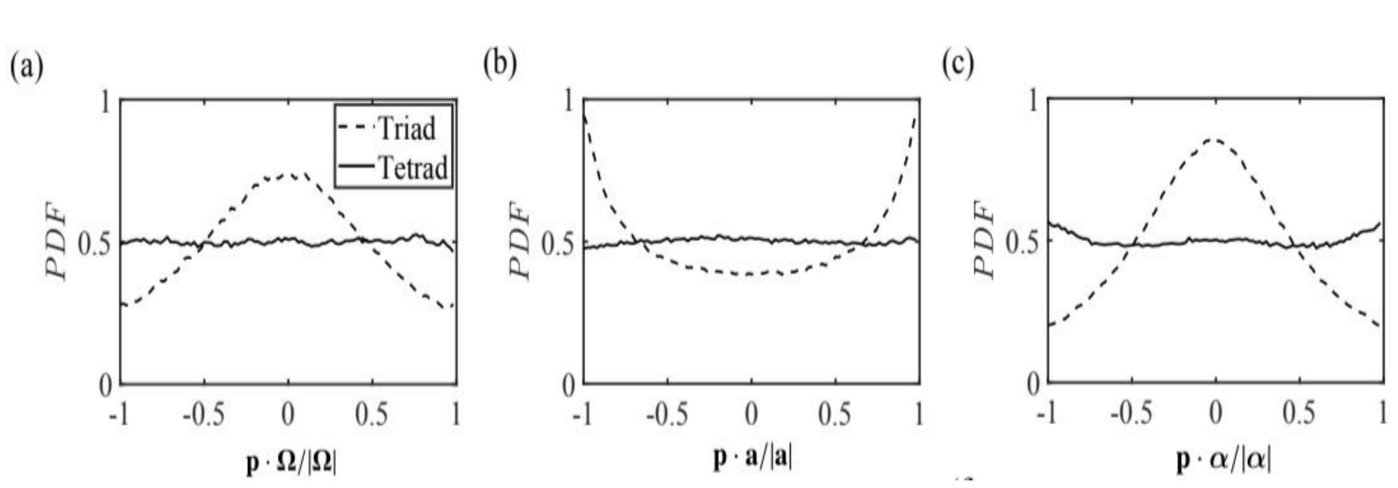


Figure 2: Probability density functions of the cosine of the angle between:
a) p and omega, b) p and a, c) p and α for a triad (dashed line) and a tetrad (solid line).

Particle Alignment in Chaotic Flows

In strain flows, particles align such that their longest dimension lines up parallel to the deformation gradient of the fluid flow. This causes rod-like particles to align with their symmetry axis parallel to the strain and for disc-like particles to align with their symmetry axis perpendicular to the strain.
The Cauchy-Green strain-rate tensor is defined by $F_{ij} = \frac{\partial x_i}{\partial X_j}$, where X is the position of a particle in the reference position, and x is the position of that particle in the final position. After multiplying F by its transpose (F^*F^T), one finds the left-hand Cauchy-Green strain tensor, $C^{(L)}_{ij} = \frac{\partial x_i}{\partial X_k}\frac{\partial x_j}{\partial X_k}$. The maximum stretch that determines the direction of alignment of large particles in strain flow is defined by the extensional eigenvector of the left Cauchy-Green strain-rate tensor, $\hat{e}_{1L}$.

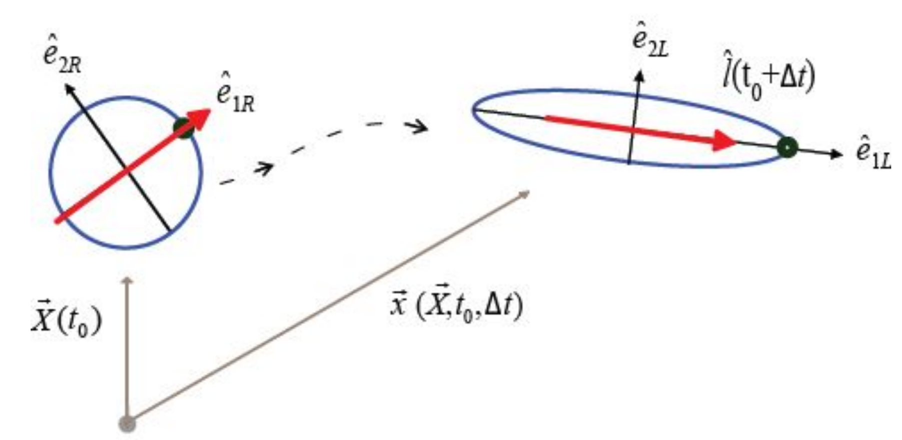


Figure 3: The Cauchy-Green strain effect on a circular 2D shape, elongated into a fiberlike shape.

Cauchy-Green strain-rate tensors are useful for understanding of turbulent flow fields because they accurately describe the alignment of disc-like particles. Since spherical particles do not have a major axis due to their isotropic nature, they do not align with any flow and orient themselves randomly. However, since discs do have a major axis, they align themselves antiparallel with $\hat{e}_{1L}$. This makes it more difficult for the particles to spin, but makes it easier for them to tumble due to conservation of angular momentum. Thus, their angular acceleration axis becomes perpendicular to the orientation axis, which is the same as the spinning axis.

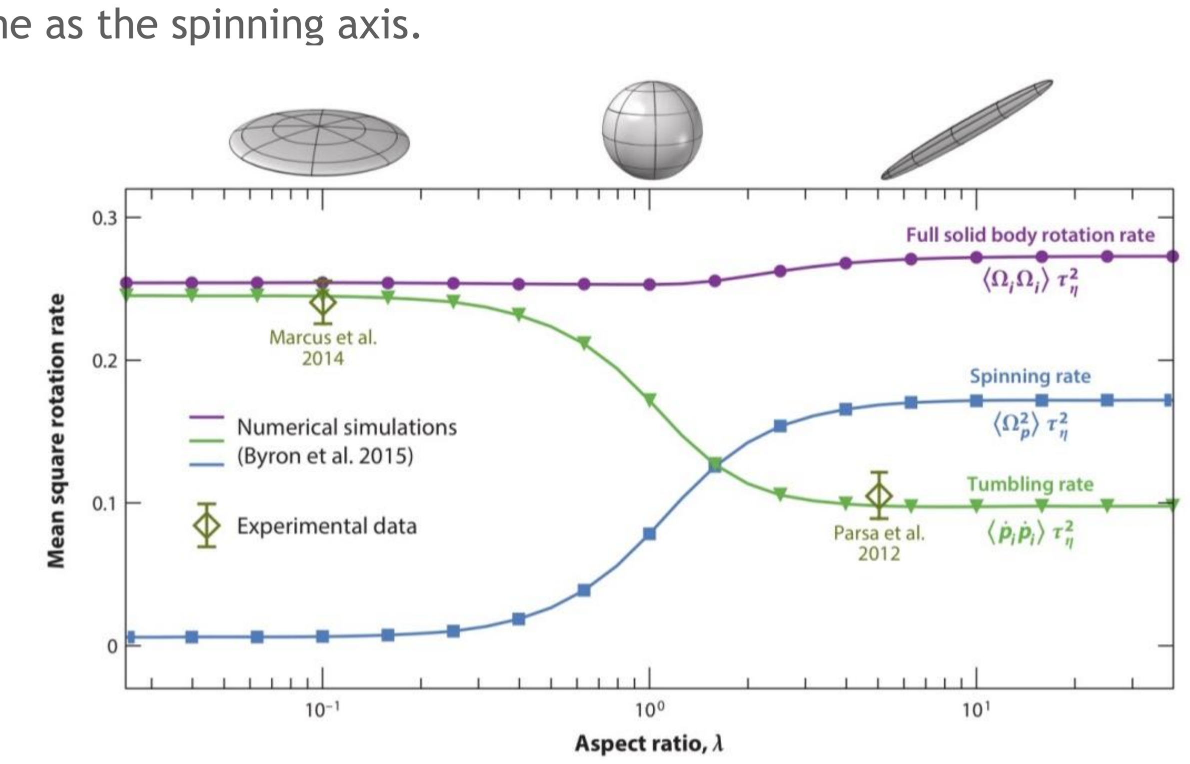


Figure 4: Mean square rotation rates of tracer particles as a function of aspect ratio in homogeneous isotropic turbulence. Also includes spinning and tumbling components. (Voth and Soldati, 2017).

Works Cited

Bardia, Hejazi. *Particle Turbulence Interactions.* 2021, Wesleyan University, Ph.D Thesis.

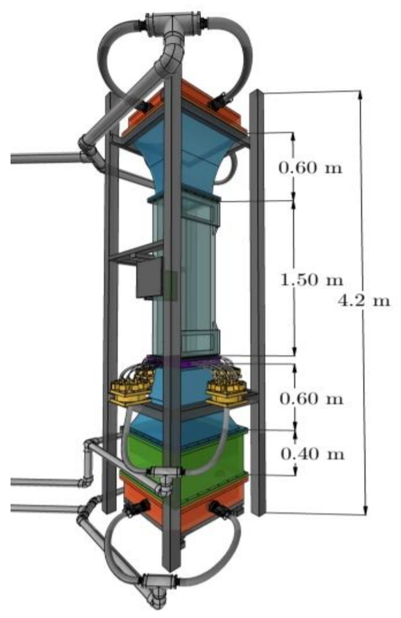
G. A. Voth and A. Soldati. Anisotropic Particles in Turbulence. Annual Review of Fluid Mechanics, 49:249-276, 2017.

Kramel, Stefan. *Non-Spherical Particle Dynamics in Turbulence.* 2017. Wesleyan University, Ph.D Thesis.

Rotation Rates of Large Particles

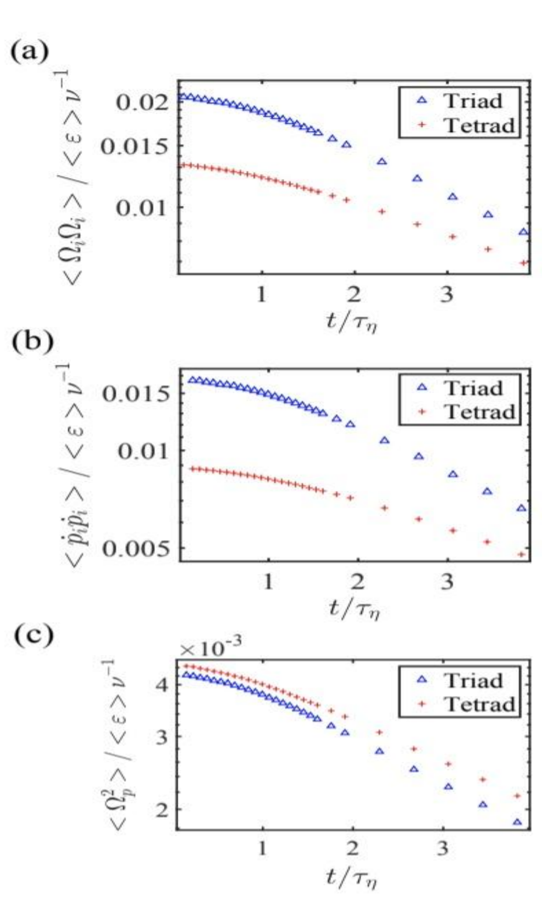
The experiments were performed using triads and tetrads, which act as discs and spheres of identical mass and volume, respectively. The experiments were performed in a vertical water tunnel to generate controlled turbulence using a random water jet-array, in the net upward direction to counteract the sedimentation effect. The motions of the particles were observed by high-speed cameras, which then sent data into a nonlinear fitting algorithm to calculate the particle's position and orientation.

Figure 5: Scaled rendering of the water tunnel which generates controlled turbulence in its central chamber (Kramel et al., 2017)



The rotation of any given particle can be defined by its spinning rate $\Omega_p = (\boldsymbol{\Omega} \cdot \mathbf{p})$ and its tumbling rate $\dot{\mathbf{p}} = (\boldsymbol{\Omega} \times \mathbf{p})$. Due to the random nature of these flows, measurements of these values are sensitive to noise. Short fit-times of the data capture these fluctuations, and longer fit-times smooth out this noise. The solid-body rotation rates of the particles plateau in short fit-times and drops over longer fit-times, demonstrating the ability to precisely measure these quantities.

Figure 6: Measured mean-square solid body rotation rate along with the components of mean square tumbling rate and mean square spinning rate for both triads and tetrads, normalized by the Kolmogorov time.
(a) The dimensionless mean square particle solid-body rotation rate ($\langle \Omega_i \Omega_i \rangle$)
(b)Dimensionless tumbling rate ($\langle \dot{p}_i \dot{p}_i \rangle$)
(c)Dimensionless spinning rate ($\langle \Omega_p^2 \rangle$)



Conclusions:

We identify five vital components of particle dynamics in turbulent flow structures, translational velocity, translational acceleration, angular velocity, angular acceleration, and particle alignment. We investigated their links with each other and what that can reveal about the flow field itself. We demonstrate the shape dependence of particle dynamics in turbulence, showing that as aspect ratio increases, spinning rate increases and tumbling rate decreases.